MAE 255 Assignement 5
Micro Mechanics Analysis Of a Lamina Using ABAQUS Finite Element Analysis

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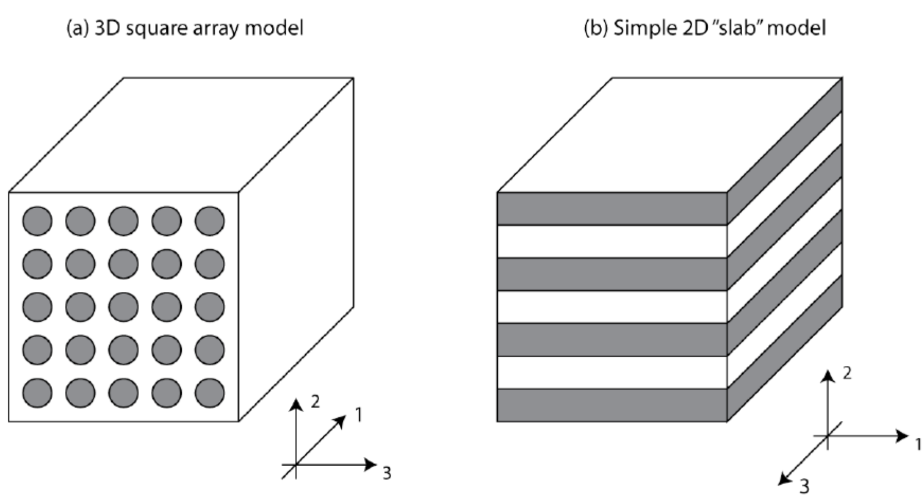
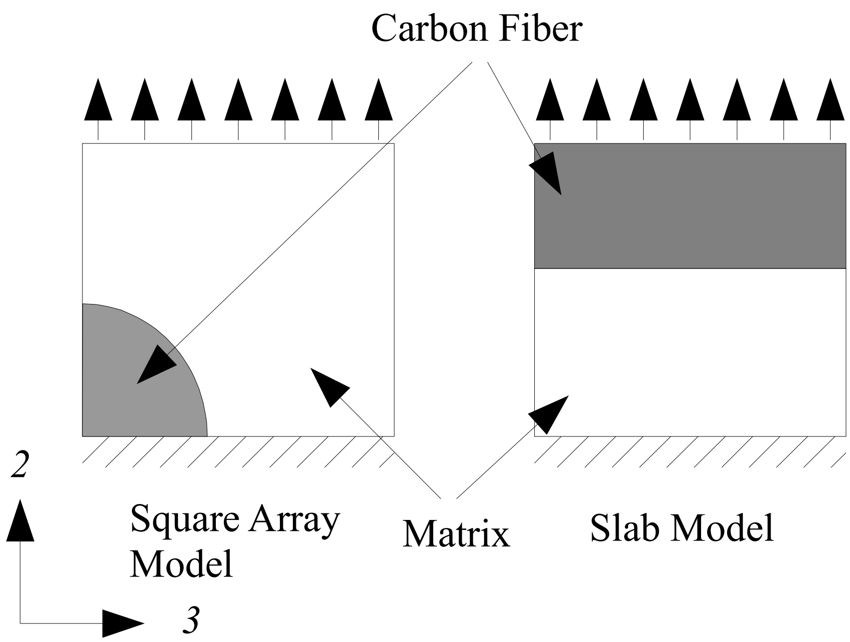
Description

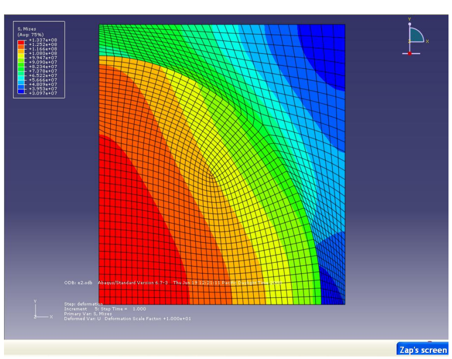
This assignement uses ABAQUS finite element analysis software to analyze the micro mechanics of a fiber reinforced lamina using a 3D square array model and a simple 2D slab model as illustrated below.

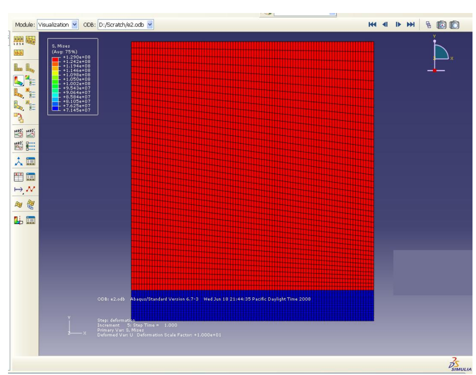


A simplified model as illustrated below was developed in ABAQUS.



Simulations of transverse loading were done on each of the models in ABAQUS as shown below.





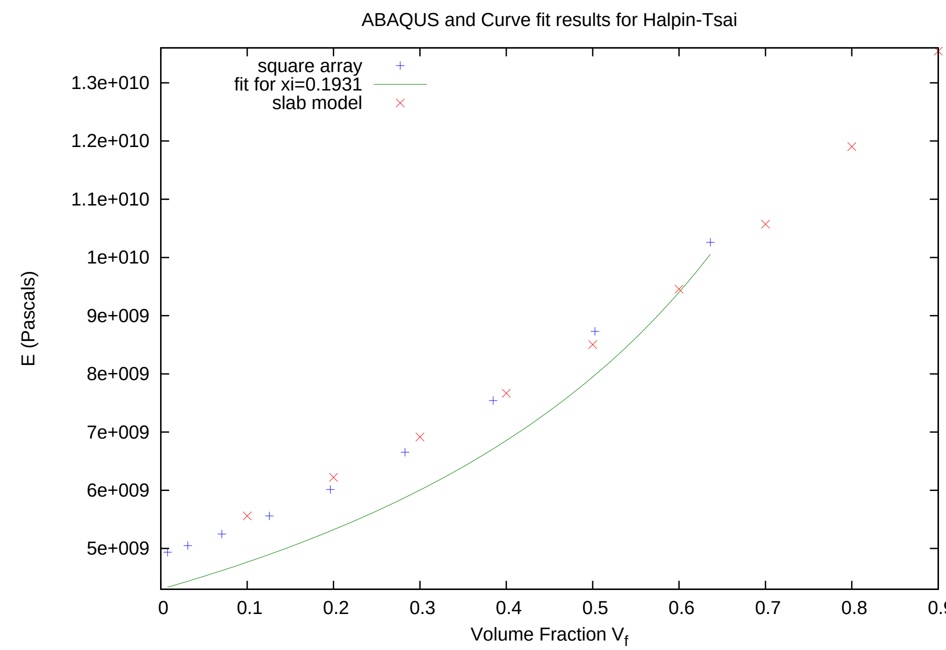
Square Array Model

Slab Model

A series of differing volume fractions V_f with a strain of 0.01 were used. Resulting force reactions were used to calculate the Halpin-Tsai Coeficient ξ for an AS4/3501-6 carbon-epoxy lamina.

1 Results

The results are plotted below for varying volume fractions V_f.



As it turns out, there is not a whole lot of difference between the slab and square array model. It is expected that the square array model would have more accurate results since it takes into account both parallel and serial loading while the slab model is purely serial.

2 Calculations

Stress was caluclated by dividing the resultant force F_2 by the area $1\,\mathrm{m} \times .01\,\mathrm{m} = .01\,\mathrm{m}^2$. The modulus of elasticity was calculated by $E = \frac{\sigma}{\epsilon}$.

The following equation was used to develop a least squares fit of the Halpin-Tsai coeficient ξ.

$$E_2 = \frac{E_m(1 + \xi\eta V_f)}{1 - \eta V_f} \tag{1}$$

Where

$$\eta = \frac{E_{f2} - E_m}{E_{2f} + \xi E_m} \tag{2}$$

The material properties are as follows
For the 3501-6 epoxy matrix

$$E_m = 4.3 \text{ GPa} \qquad \nu_m = 0.35$$

For the AS4 Carbon fiber

$$E_{2f} = 15 \text{ GPa} \qquad \nu_f = 0.20$$

The reaction forces from ABAQUS for the square array model were

Square Array Model Force Reactions

F_2	V_f
493653	0.007854
504880	0.0314159
524933	0.0706858
555969	0.1256637
601281	0.1963495
665427	0.2827433
754068	0.3848451
873063	0.5026548
1026020	0.6361725

and the Output for the slab model are

Slab Model Force Reactions

F_2 slab	V_f
555884	555884
622165	622165
691609	691609
766767	766767
850377	850377
945722	945722
1057100	1057100
1190400	1190400
1354400	1354400

The Halpin-Tsai Coeficient was found from the Square array model from a linear regression fit developed below.

$$
\begin{aligned}
E_2 =& \frac{E_m(1+\xi\eta V_f)}{1-\eta V_f} \\
E_2(1-\eta V_f) =& E_m(1+\xi\eta V_f) \\
E_2 - E_2\eta V_f =& E_m(1+\xi\eta) \\
=& E_m + E_m\xi\eta) \\
E_2 - E_m =& E_2\eta V_f + E_m\xi\eta \\
\underbrace{\begin{bmatrix} E_{2_{(1)}} - E_m \\ \vdots \\ E_{2_{(n)}} - E_m \end{bmatrix}}_{\boldsymbol{b}} =& \underbrace{\begin{bmatrix} E_2 V_{f_{(1)}} & E_m \\ \vdots & \vdots \\ E_2 V_{f_{(1)}} & E_m \end{bmatrix}}_{\mathsf{A}} \underbrace{\begin{bmatrix} \eta \\ \xi\eta \end{bmatrix}}_{\boldsymbol{x_{LS}}}
\end{aligned}
$$

From the least squares fit we get

$$\boldsymbol{x}_{LS} = (\mathsf{A}^T \mathsf{A})^{-1}\mathsf{A}^T\boldsymbol{b}$$

This yields

$$\begin{bmatrix} \eta \\ \xi\eta \end{bmatrix}_{LS} = \boldsymbol{x}_{LS}$$

ξ is the calculated by

$$\xi = \frac{\xi\eta}{\eta}$$

The above fit was calculated using Octave (a Matlab Clone). This value of ξ was used in equations (1) and (2) for the plotted curve in the results.

3 Computer Code

The following Matlab/Octave code was used to curve fit and plot the results.

```
% MAE255 assignment 5 Extra Credit
%Matrix material properties
E_m=4.3e+9 % Pascals
nu_m=0.35
%Fiber Properties
E_2f=15e+9 %Pascals
nu_12f=0.20
```

```
%length and width of cell
l=10^(-2)
%loading
delta_l=.0001
strain=delta_l/l
%Data from ABAQUS square array model
F_2=[493.653E+03 504.88E+03 524.933E+03 555.969E+03 601.281E+03 665.427E+03 754.068E+
%radius of fiber
r=[1 2 3 4 5 6 7 8 9]'*10^(-3)
%Data from slab model
F_2_slab=[555.884 622.165 691.609 766.767 850.377 945.722 1057.1 1190.4 1354.4]*10^3
% Volume fractio from slab model
V_f_slab=[.1 .2 .3 .4 .5 .6 .7 .8 .9]
stress_slab=F_2_slab/l
E_2_slab=stress_slab/strain

%
%stress for slabe model
area=l*1
stress=F_2/area
E_2=stress/strain
%calculation of V_f fiber volume fraction
V_f=(1/4*pi*r.^2/(l^2))
%linear regression fit for square cell model
b=E_2-E_m
A=[E_2.*V_f ones(length(V_f),1)*E_m]
x_LS=inv(A'*A)*A'*b
% calculation of Halpin-Tsai coeficient xi
eta=x_LS(1)
xi_eta=x_LS(2)
xi=xi_eta/eta
%xi=.5
%calculation E2 curve fit
V_f_calc=[min(V_f):(max(V_f)-min(V_f))/100:max(V_f)]
E_2_calc=E_m*(1+xi*eta*V_f_calc)./(1-eta*V_f_calc)

plot(V_f,E_2,'+',V_f_calc,E_2_calc,V_f_slab,E_2_slab,'x')
title('ABAQUS and Curve fit results for Halpin-Tsai')
xlabel('Volume Fraction V_f')
ylabel('E (Pascals)')
```

```
xi_st=['fit for xi=',num2str(xi)]
legend('square array',xi_st,'slab model',2)
%print -depsc mae255_asgn5_results.eps
```